Optimal Transmission Policies for an Energy Harvesting Transmitter with Processing Cost

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Motivation

- Wireless nodes are battery limited.
- Replacing batteries may be expensive or inconvenient.
  - Solution: Energy harvesting.

- Harvested energy is typically low due to physical and technological limitations.
- Goal:
  - Increase battery life.
  - Ensure reliable operation.
- Challenge: Management of harvested energy.
Energy Cost of Transmitter

- Energy consumed for communication includes:
  - Transmission energy.
  - Processing energy from operation of mixers, filters, converters etc.

- Transmission and processing energy can be more or less significant depending on:
  - Communication range.
  - Processing circuitry.
Consider single energy arrival and a static channel gain:

- Without processing cost, increasing transmission duration and lowering power increases throughput.

- With processing cost, lowering transmission power does not increase throughput after a certain point.

- Processing energy dominates consumed energy when $p < v^*$. 
- Transmission energy dominates consumed energy when $p > v^*$. 
Consider:

- Single energy arrival $E$ at $t = 0$.
- Static channel gain $h$.
- Constant processing cost $\epsilon$.
- Total transmission duration $\Theta$.
- No transmission deadline.

Maximum throughput is given by

$$\max_{\Theta, \nu: \Theta(\nu + \epsilon) \leq E} \frac{\Theta}{\nu} \log(1 + h\nu),$$

where the optimal transmission power $\nu^*$ satisfies

$$\frac{1}{h + \nu^*} = \frac{1}{\epsilon + \nu^*} \log(1 + h\nu^*).$$
When there is a transmission deadline $T$, the optimal transmission policy satisfies

$$(p^*, \Theta^*) = \begin{cases} 
\left( v^*, \frac{E_0}{v^*+\epsilon} \right) & \text{if } T \geq \frac{E_0}{v^*+\epsilon}, \\
\left( \frac{E_0}{T} - \epsilon, T \right) & \text{if } T < \frac{E_0}{v^*+\epsilon}.
\end{cases}$$

Single Energy Arrival and Multiple Fading Levels:

Glue pouring

An adaptation of classical water-filling algorithm when processing cost is not negligible.

- Processing cost $\Rightarrow$ Optimal transmission policy is bursty.
- Single energy arrival and multiple fading levels $\Rightarrow$ Glue pouring.
Glue Pouring versus Water-Filling

Glue pouring:

Water-filling:
Glue Pouring versus Water-Filling

Glue pouring: 

\[ \frac{1}{h_1} \quad \tau_1 \quad \frac{1}{h_2} \quad \tau_2 \]

Water-filling: 

\[ \frac{1}{h_1} \quad \tau_1 \quad \frac{1}{h_2} \quad \tau_2 \]
Glue Pouring versus Water-Filling

Glue pouring:

\[ \frac{1}{h_1} \rightarrow t \quad \frac{1}{h_2} \quad v_1^* \]

Water-filling:

\[ \frac{1}{h_1} \rightarrow t \quad \frac{1}{h_2} \quad v_2^* \]
Glue Pouring versus Water-Filling

Glue pouring:

\[ \frac{1}{h_1} \quad t \quad t_1 \]

\[ v_1^* \]

\[ \frac{1}{h_2} \]

Water-filling:

\[ \frac{1}{h_1} \quad t \quad t_1 \]

\[ v_2^* \]

\[ \frac{1}{h_2} \]
Glue Pouring versus Water-Filling

Glue pouring:

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Glue pouring:

Water-filling:
Transmitter harvests energy $E_i$ at times $t_{e,i}$, $i = 0, \ldots, n$. 
Energy stored in battery of capacity $E_{\text{max}}$.
AWGN channel gain $h(t)$ changes at times $t_{f,i}$, $i = 0, \ldots, m$.
Rate-power function
- Shannon capacity $\frac{1}{2} \log(1 + h(t)p(t))$, where $p(t)$ is transmission power at time $t$.

Transmitter consumes constant processing energy $\epsilon$ joules per unit time when transmitting.
Optimal transmission policy must satisfy following constraints.

- **Energy causality:**

  \[ E(t) \leq \sum_{i:0 \leq t_i < t} E_i, \quad \forall t \in [0, T]. \]

- **Battery size constraint:**

  \[ \sum_{i:0 \leq t_i \leq t} E_i - E(t) \leq E_{\text{max}}, \quad \forall t \in [0, T]. \]

- **Data causality:**

  \[ B(t) \leq \sum_{i:0 \leq t_i < t} B_i, \quad \forall t \in [0, T]. \]
System Model

- **Offline optimization:** Energy and data arrivals and fading profiles are known at transmitter at time $t = 0$.
  - Can model predictable systems.
  - Provide upper bound on performance.

- **Online optimization:** A casual knowledge of the underlying stochastic processes is assumed.
  - More realistic model.
  - A near optimal low-complexity transmission policy based on the heuristics gained from the offline optimization.
System Objective

- Throughput maximization
  - Transmitter has sufficient data in its data buffer.
  - **Goal:** Maximize throughput, total transmitted data by deadline $T$. 
Throughput Maximization

Combine all energy arrivals and channel gain changes in a single time series $t_0 = 0 < t_1 < \cdots < t_{N-1} < T$.

Time interval between two events is called an epoch.

For epoch $i$,

- duration: $\tau_i = t_i - t_{i-1}$
- channel gain: $h_i$
- transmit power: $p_i$
- transmission duration: $\Theta_i$
The throughput maximization problem:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{N} \frac{\Theta_i}{2} \log(1 + h_i p_i) \\
\text{subject to} & \quad \sum_{j=1}^{i} (\Theta_j (p_j + \epsilon) - E_{j-1}) \leq 0, \quad i = 1, \ldots, N, \\
& \quad \sum_{j=1}^{i+1} E_{j-1} - \sum_{j=1}^{i} \Theta_j (p_j + \epsilon) \leq E_{\text{max}}, \quad i = 1, \ldots, N, \\
& \quad 0 \leq \Theta_i \leq \tau_i, \quad \text{and} \quad 0 \leq p_i, \quad i = 1, \ldots, N,
\end{align*}
\]

where the first constraint is due to energy causality, the second constraint is due to finite battery size.

- It can be transformed to convex optimization problem by defining a new variable \( \alpha_i \triangleq \Theta_i p_i \).
Optimal transmission power $p_i^*$ satisfies the following based on $\Theta_i^*$:

- If $\Theta_i^* = 0$, then $p_i^* = 0$.
- If $0 < \Theta_i^* < \tau_i$, then $p_i^* = v_i^*$.
- If $\Theta_i^* = \tau_i$, then

$$p_i^* = \frac{1}{2 \sum_{j=i}^{N} (\lambda_j - \mu_j)} - \frac{1}{h_i}$$

where $p_i^* > v_i^*$. 
Remark

From complementary slackness conditions;

- $\lambda_i > 0$ and $\mu_i = 0 \Rightarrow$ battery depletes.
- $\lambda_i = 0$ and $\mu_i > 0 \Rightarrow$ battery full.
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Remark

Whenever the sum of the inverse channel gain and the optimal power level increases (decreases) from one epoch to next, battery must be empty (full).
Allocate harvested energy to epochs starting from the last non-zero energy packet.
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Transfer at most $E_{max} - E_{j-1}$ amount of energy to epoch $j$. 
\[ \epsilon = 0, \text{ throughput} = 2.11 \text{ nats}. \]
\( \epsilon = 1 \, \mu W \), throughput= 1.39 nats.

- Optimal transmission policy is bursty.
- Due to finite capacity battery, optimal policy allocates energy to epoch two even though it has the worst channel gain.
Optimal throughput decreases as the processing energy cost increases.
We identified optimal transmission policy for throughput maximization problem.

We provided a convex optimization formulation and ‘directional glue pouring’ interpretation for the transmission power.

Ongoing work:

- Energy maximization problem.
- Transmission completion time minimization problem.
- Extensions to parallel channels (e.g. in frequency).
