Algorithms for Determining File Distribution in Networks with Multimedia Servers *

Jeong-dong Ryoo and Shivendra S. Panwar
Polytechnic University
Five Metrotech Center, Brooklyn, NY 11201, USA

Abstract

In this paper, we consider an optimization problem in networks with storage servers for providing multimedia service. The design involves assigning communication link capacity, sizing the multimedia servers and distributing different types of content at each server, while guaranteeing an upper limit on the individual end-to-end blocking probability. We present optimization algorithms to obtain an optimal solution. Under a linear cost structure, our numerical investigations consider different scenarios that might be helpful in understanding how to distribute multimedia content for a cost-optimized solution.

1 Introduction

In the recent past, many researchers have investigated the issues related to the delivery of interactive video service in networks with video servers. Among them, there has been much interest in dealing with network design issues related to video-on-demand service [1].

The types of delivered service need not be confined to video-on-demand, but can be extended to a variety of multimedia services, e.g., digitized voice, high quality audio, and interactive video. The characteristics of multimedia service, such as required bandwidth for delivering a continuous stream of data, user access probability requirements, and holding times, influence the design of a cost-optimized architecture. This also further complicates the problem to be addressed in determining file distribution in the network.

In this paper, we consider an optimization problem in networks with storage servers for providing multimedia service. We assume that the connection blocking probability for an end user's request to obtain a desired service is the performance criterion of interest. Unlike other work that focused on the proportion to be placed at local or central servers, our design involves assigning communication link capacity, sizing the multimedia servers and distributing different types of service content at each server, while guaranteeing an upper limit on the individual end-to-end blocking probability.

*Supported in part by the New York State Center for Advanced Technology in Telecommunications and Distributed Information Systems(CATT), Polytechnic University.

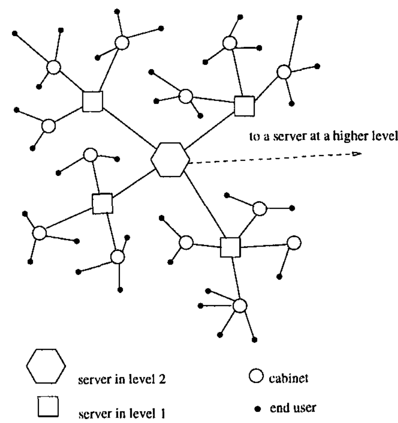


Figure 1: Network reference model

First, we start with a model formulation for providing multimedia service. After we formally define our optimization problem, we discuss how we can use approximations to compute the end-to-end blocking probability to reduce optimization time. In Section 3, we present two algorithms to obtain an optimal solution, based upon the properties of the tradeoffs between communication link cost and storage server cost. Under a linear cost structure, our numerical investigations consider different scenarios that might be helpful in understanding how to distribute multimedia content for a cost-optimized solution.

2 Model Formulation

2.1 A structure for multimedia service networks

Consider a network with an arbitrary tree topology as shown in Fig. 1. Every residential customer is connected to a point, called a distribution cabinet in Fig. 1. This point corresponds to an *Optical Network Unit*(ONU) in an optical distribution network, a switch in the *Central Office*(CO) of telephone network, or a headend in a cable network, depending on the access network technology. When this concentration point is absent and a customer request goes to a multimedia server directly, we can easily reformulate a new cost function and performance evalua-

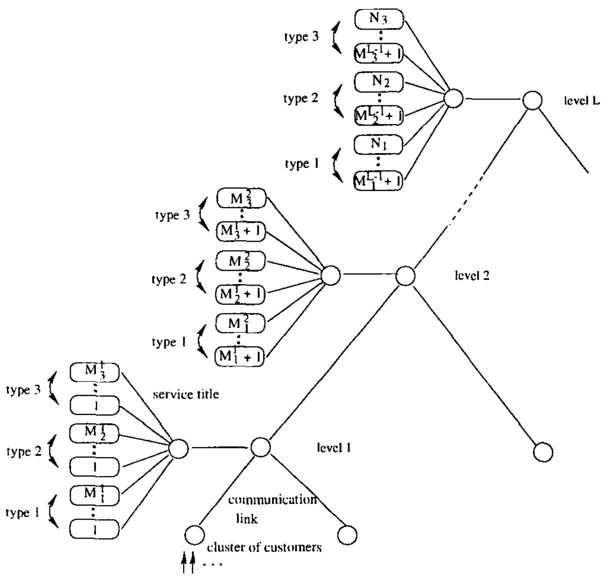


Figure 2: File distribution for predetermined server selection

tion formula. Each node above this point is a potential site for a multimedia server. Multimedia servers can be placed in any node at any level of a tree network.

Once we focus on end-to-end or call blocking probability as a measure of grade of service, the two most important parameters of a multimedia server are the bandwidth of the I/O controller that is attached to the network and the maximum number of simultaneous streams that a server can support for a specific title. Thus a multimedia server is assumed to perform admission control for a new request based upon the simultaneous availability of the output bandwidth in the server and the requested file.

In this paper, the number of streams that can access a specific title is assumed to be limited by the "number of copies" of the particular file. What we mean by the "number of copies" is the number of independent streams that can be supported simultaneously for the requested title, which in general may exceed the actual number of physical copies.

A request for a multimedia service is assumed to be assigned to a predetermined server that contains the requested file. The content files are distributed among the servers and each server contains a subset of the available files. We assume that files are placed, starting with the highest level server, in inverse order of their popularity. Thus by placing the most popular titles closest to the users, the communication cost can be reduced, and the incremental storage cost is less due to the multiplexing gain. Through numerical investigations, we previously observed that this assumption of placing the most popular titles to low level servers appears to be valid for our network design problem under the end-to-end blocking constraint [2].

In Fig. 2, we show the file distribution for predetermined server selection in the case of an L level tree network. w_l denotes the number of nodes at level l of the tree, $l = 0, 1, \dots, L$, where we define $w_L = 1$. Three types of services are available in this example. Types of services are classified according to their bandwidth requirements and mean holding time characteristics. N_i denotes the number of available titles for service type $i \in \mathcal{I}$, where $\mathcal{I}$ is the set of service types, in increasing bandwidth order. The bandwidth of service type i is denoted by d_i. We will index the available titles from 1 to N_i, starting with the most popular file in type i. Let M_i^l, the *cutoff index*, be the index of the least popular type i file stored in a level l server. Therefore, the files ranging from the $(M_i^{l-1}+1)$th most popular title to the M_i^lth one will be placed in a multimedia server in level l, where $M_i^0 = 0$. The required storage capacity m_l in a level l server is $\sum_{i\in\mathcal{I}}\sum_{j=M_i^{l-1}+1}^{M_i^l} t_{ij}^l b_{ij}$, where t_{ij}^l is the number of copies of the jth popular file of type i in multimedia server at level l and b_{ij} is the required space for storing one copy of the jth popular file of type i.

2.2 Optimization model

The objective of the design is to minimize the overall cost, which is the sum of the costs of multimedia servers and communication link costs. For reasons that we will discuss in Section 3, the minimization of the total cost expression below will be divided into the two steps, as denoted by the inner and outer minimizations:

$$\min_{\{M_i^l\}}\{\min_{\substack{\{k_l\}\\ \{s_l\}\\ \{t_{ij}^l\}}}\{\sum_{l=0}^{L-1} w_l\psi_l(k_l) + \sum_{l=1}^{L} w_l\phi_l(s_l, \sum_{i\in\mathcal{I}}\sum_{j=M_i^{l-1}+1}^{M_i^l} t_{ij}^l b_{ij})\}, \quad (1)$$

with constraints

$$\begin{array}{rcl} B_{ij}(\underline{k}, \underline{s}, \mathbf{t}) & \leq & \hat{B_{ij}} \quad \forall i, j \\ k_l & \geq & 0 \quad \forall l \\ s_l & \geq & 0 \quad \forall l \\ t_{ij}^l & \geq & 0 \quad \forall i, j, l \\ M_i^1 \leq M_i^2 \leq \dots & \leq & M_i^L \quad \forall i \end{array}$$

The communication link cost, $\psi_l(k_l)$, is the cost of the link between level l and $l+1$ with capacity k_l. $\phi_l(s_l, m_l)$ represents the cost of a multimedia server at level l with output bandwidth s_l and storage capacity m_l. $B_{ij}(\underline{k}, \underline{s}, \mathbf{t})$ is the end-to-end blocking probability for the jth popular file of type i, which is constrained to be less than $\hat{B}_{ij}$. Depending on the values of the M_i^l's, some links and servers may be eliminated from the network(for example, when $M_i^{l-1} = M_i^l$ for all i, level l servers don't exist in final solution).

The total cost expression in (1) assumes symmetric traffic requirements, i.e., each cluster of users has the same demand characteristics for each title. Consequently, the results are obtained for a balanced tree, or in degenerate cases, a forest of balanced trees, all the links at any particular level having the same capacity. With minor modifications, this symmetric assumption can be loosened at the expense of increasing the decision variables.

Requests for a service title arrive according to a Poisson process. The route for the service title consists of communication links from the customer cluster to the server which provides the service title, the I/O of the server and a copy of the required title. Each element in the route has finite units of bandwidth, which is shared among different classes of traffics. A request takes predetermined units of bandwidth at each element on the route simultaneously, provided it is not blocked by any of the elements in the route. The length of time that the title is accessed, i.e., the service duration, has a general distribution with finite mean.

Since it is hard to compute the end-to-end blocking probability even for a moderate sized network, we consider two approximations: the reduced load approximation and the summation approximation. Details of these approximations can be found in [3], under the assumption that all service requests require the same bandwidth. Among several methods for calculating single element loss probability [4], we decided to use the Uniform Asymptotic Approximation(UAA) [5] for calculating the single element blocking probability for each type of traffic. In addition to its capability of handling real values of capacity, which is useful for our optimization, the UAA is quite accurate for calculations of single link blocking probabilities for services with heterogeneous bandwidths. Another attractive feature is that the complexity of computation does not increase with the link capacity.

From our numerical experience, even though the difference between the reduced load and summation approximations increases with higher loads and bandwidth requirements, the summation approximation provides a tight upper bound on the reduced load approximation at the loading region of 1% end-to-end blocking probability [2]. Generally, the repeated substitution method is used to get a set of converged solutions for the reduced load approximation.The number of iterations in the repeated substitution depends on not only the stopping criterion of the iteration but also the values of traffic loads and system capacities. However, the summation approximation gives the solution in one iteration. Consequently, the summation approximation has an advantage in computation time, at the cost of somewhat reduced accuracy.

3 Optimization Method

In this section, we consider an optimization technique for solving the problem introduced in Equation (1). To avoid dealing with a nonlinear integer programming problem for the inner optimization, we assume a continuous cost function and allow all the decision variables except the M_i^l's to be real. Since it is difficult to decide the value of the cost function at non-integer values of M_i^l's, we keep the decision variables of the outer minimization as integers.

From our computational experience using the Matlab optimization package, which has an implementation of sequential quadratic programming, the CPU running time for one inner minimization routine(Inner_Min) under the reduced-load approximation of the end-to-end blocking constraint is about five times as large as the time under the summation approximation. However, using the solution from the summation approximation as a starting point, the iterations under the reduced load approximation terminates in about half the time that it takes from an arbitrarily chosen original initial point. Moreover, from extensive numerical investigation, we believe that the choice of the approximation schemes does not affect the optimal values of M_i^l's. Thus, the benefit of the optimization time using the summation approximation becomes more significant for the outer minimization.

As we saw in [2], once the cost decreases by moving the most popular title from a higher level server down to lower level servers, the cost keeps decreasing until reaching the optimal cutoff index. After passing the optimal cutoff index, the cost increases as we further increase the value of M_i^l. This indicates that the optimization problem has a well-behaved cost structure with respect to the M_i^l's. Thus we suggest the following two alternative procedures for our optimization problem. In both procedures, the summation approximation is used for all but the last step.

Procedure 1(optimize each service independently)

$J_{old} = \infty$ (J is the total cost)
$M_i^L = N_i$, $M_i^l = 0$ for all i and $1 \leq l < L$
for $i = |\mathcal{I}|$ down to 1 (start with largest bandwidth service)
  compute J_{new}, cost from inner minimization subroutine Inner_Min
  do the following routine while $J_{new} < J_{old}$ (search for optimal cutoff indices for service i)
    $J_{old} = J_{new}$
    $\hat{M}_i^l = M_i^l$ for all l
    for $l = 1$ to $L-1$ (calculate cost for new cutoff indices)
      for $j = l$ to $L-1$
        $M_i^j = \max\{M_i^l + Z_i^l, M_i^j\}$ (Z_i^l is a step size.)
      compute J^l, the cost from Inner_Min
      $M_i^l = \hat{M}_i^l$ for all l
    end
    for $l = 1$ to $L-1$ (pick new point in "optimum" direction)
      if $J^l < J_{old}$
        $M_i^l = M_i^l + Z_i^l$
      else
        $M_i^l = \max\{0, M_i^l - Z_i^l\}$
        $Z_i^l = \max\{1, \lfloor \frac{Z_i^l}{2} \rfloor\}$
    end
    for $l = 1$ to $L-1$ (make cutoff indices consistent)
      $M_i^l = \max\{M_i^l, M_i^{l-1}\}$
    compute J_{new} for new M_i^l's using Inner_Min
  end(do)
end(for)
Perform final inner minimization using reduced-load approximation

Procedure 2(descent direction-based search)

$J_{old} = \infty$
$M_i^L = N_i$, $M_i^l = 0$ for all i and $1 \leq l < L$
compute J_{new}, the cost from Inner_Min
do the following routine while $J_{new} < J_{old}$
  $J_{old} = J_{new}$
  $\hat{M}_i^l = M_i^l$ for all i and l
  for $i = 1$ to $|\mathcal{I}|$ (compute costs in all directions)
    for $l = 1$ to $L-1$

```
            for j = l to L - 1
                M_i^j = max{M_i^l + Z_i^l, M_i^j}
            compute J_i^l, the cost from Inner_Min
            M_i^l = M_i^l for all l
        end
    end
    for i = 1 to |I|        (pick new point in descent direction)
        for l = 1 to L - 1
            if J^l < J_old
                M_i^l = M_i^l + Z_i^l
            else
                M_i^l = max{0, M_i^l - Z_i^l}
                Z_i^l = max{1, floor(Z_i^l / 2)}
        end
        for l = 1 to L - 1        (make cutoff indices consistent)
            M_i^l = max{M_i^l, M_i^{l-1}}
    end
    compute J_new for new M_i^l's using Inner_Min
end(do)
Perform final inner minimization using reduced-load approximation
```

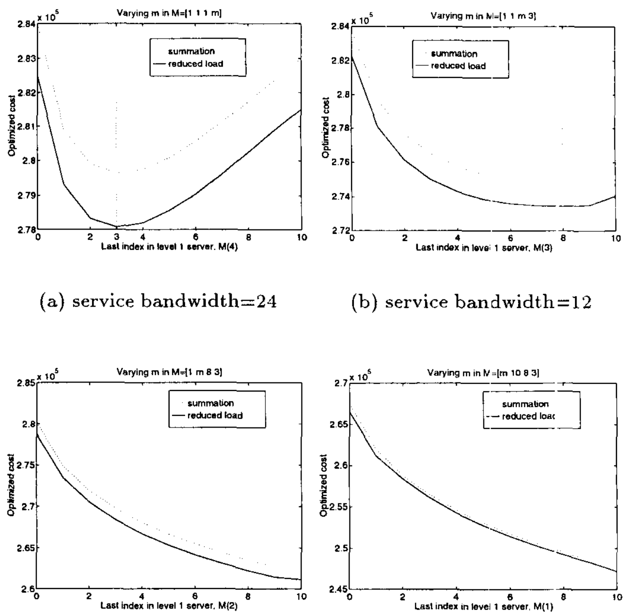
Z_i^l is initially an arbitrarily chosen positive integer value, which is adjusted during iterations as indicated above, and corresponds to a step size. An exhaustive search based outer minimization that finds a set of the optimal cutoff indices has complexity of $\mathcal{O}(\prod_{i=1}^{|\mathcal{I}|}(N_i)^{L-1})$. Using Procedure 1, the complexity is at most $\mathcal{O}(\sum_{i=1}^{|\mathcal{I}|}(N_i)(L-1))$. For Procedure 2, the inner "for" loop takes $|\mathcal{I}|(L-1)$ iterations, while the outer "do" loop iterates $\mathcal{O}(\max N_i)$ times in most cases from our computational experience. In practical situations, L tends to be a small value, such as two or three. The number of titles of each type of service N_i could be a much larger value, a few hundred or thousands. Also as the types of services increase, $|\mathcal{I}|$ gets larger. Thus the gain with the proposed algorithm is considerable, and allows practical sized problems to be solved.

4 Numerical Investigations

In this paper, we consider a two-level system with one root server and several local servers. The constraint on the individual end-to-end blocking probability is assumed to be less than or equal to 0.01 for all i and j. For each type of service i, the user access probability for title j, p_j, is assumed to be the Zipf distribution with parameter $\theta = 0.271$ [6]. In both cases, we set $N = [10 \quad 10 \quad 10 \quad 10], w_1 = 8, w_0 = 32$. Four different types of services are assumed to be available, and their bandwidth requirements, $\boldsymbol{d} = [1 \quad 2 \quad 12 \quad 24]$. This could correspond, for example, to service ranging from voice at 64kbps to compressed video at 1.5Mbps. More numerical results representing other cases are presented in [2].

4.1 Case 1

This case deals with the situation that the total offered load for type i is the same for all types of services, i.e., $\sum_{j=1}^{N_i} \rho_{ij} d_i = 120$ for all i, where ρ_{ij} is the traffic intensity(the product of arrival rate and service duration) for



(a) service bandwidth=24 (b) service bandwidth=12

(c) service bandwidth=2 (d) service bandwidth=1

Figure 3: Optimized costs with $c = 5$

the jth popular file of type i. Additionally, the mean service duration for all types of service is assumed to be the same. The cost function is assumed to be linear with coefficients shown in Table 1. In order to study the tradeoff between communication cost and storage cost, we chose the coefficient of the link cost between a level 1 server and a level 2 server, c, to be a variable parameter.

	Case1	Case2
one copy of type 1 file	4	1
one copy of type 2 file	8	6
one copy of type 3 file	48	84
one copy of type 4 file	96	600
one unit of server's output bandwidth	1	3
one unit of link bandwidth between level 0 and 1	5	10
one unit of link bandwidth between level 1 and 2	c	c

Table 1: Cost coefficients for Case 1 and Case 2

When $c = 5$, the effect of different file allocation schemes on total cost is shown in Fig. 3. The figure also shows the progress of algorithm 1. Every point in a curve is the optimized cost from the inner minimization of our optimization problem (1). We first vary the value of M_1, while the values of other three M_i's are fixed at 1. The best value from this step is $M_1 = 3$, as shown in Fig. 3(a). After finding the best value of M_1, we vary the next parameter, M_2. The same routine is performed until the best values of all M_i's are obtained. Thus the solution for the outer minimization of (1) is $\boldsymbol{M} = [10 \quad 10 \quad 8 \quad 3]$. Up to this point all the optimization routines are performed using the summation approximation. Finally, to get the optimal values of decision variables in inner minimization, we perform the final step with the reduced load approximation. In this figure, in order to provide better understanding, we draw the curves for both approximation methods and all the possible value of M_i along intermediate steps of the algorithm. Note that the reduced load and summation approximations find identical optimal values for the cutoff indices.

	M_i (Procedure 1/Procedure 2)				
c	Type 1	Type 2	Type 3	Type 4	Optimized Cost
0.01	4/4	0/1	0/0	0/0	204146/204012
0.1	5/5	1/1	0/0	0/0	205515/205515
0.3	7/7	2/2	0/0	0/0	208553/208553
0.5	10/10	6/4	0/0	0/0	211423/211303
1	10/10	8/8	0/0	0/0	219682/219683
2.5	10/10	10/10	0/2	0/0	234172/232776
3	10/10	10/10	3/3	1/1	236601/236601
4	10/10	10/10	5/5	2/2	243074/243074
5	10/10	10/10	8/8	3/4	247726/247619
6	10/10	10/10	10/10	5/9	250382/249193
7	10/10	10/10	10/10	10/10	250186/250186

Table 2: Optimal values of the cutoff indices for Case 1

When we vary the value of c, the impact on the optimal values of M_i's is shown in Table 2. Procedure 1 is used for the value before a slash mark. The results from Procedure 2 are shown after the slash mark. From the table, we can see that the files with smaller bandwidth requirement are distributed among multiple local servers, while one central server tends to keep the files with larger bandwidth requirements. The two procedures find the same cutoff indices in most cases. However, Procedure 2 gives slightly better results at the expense of higher optimization time.

4.2 Case 2

In this case, we assume that the sum of traffic intensities to the files with the same type of service, $\sum_{j=1}^{N_i} \rho_{ij} = 20$, is the same for all the types of services i's. The service time for the four types of service are assumed to be 3 min, 9 min, 21 min, and 75 minutes, respectively. The cost coefficients for unit storage for different service types are proportional to the product of their holding times and bandwidth.

The cost function is assumed to be linear with the coefficients listed in Table 1. The result from Procedure 1 is shown in Table 3. Again, we see that the central server keeps the files with the larger bandwidth requirements.

	M_i			
Coefficient, c	Type 1	Type 2	Type 3	Type 4
0.1	10	7	0	0
1	10	10	0	0
2	10	10	3	0
4	10	10	7	0
8	10	10	10	0
12	10	10	10	3
15	10	10	10	5
18	10	10	10	10

Table 3: Optimal values of the cutoff indices for Case 2

5 Conclusion

In this paper, we consider an optimization problem in networks with storage servers for providing multimedia service under a constraint on the individual end-to-end blocking probabilities. We presented two algorithms to find the best file distribution schemes as well as optimize the size of the elements in a network with multimedia servers. A well-managed multimedia file distribution scheme reduces storage cost and communication cost, making the service more affordable. This optimization can be used in content management as the demand characteristics of files vary with time, as well as in the initial design phase.

References

[1] Jean-Paul Nussbaumer, Baiju V. Patel, Frank Schaffa, and James P.G. Sterbenz, "Networking requirements for interactive video on demand," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 5, pp. 779–787, June 1995.

[2] Jeong-dong Ryoo and Shivendra S. Panwar, "File distribution schemes in networks with multimedia server: Multirate case," *CATT - Technical Report, Polytechnic University*, 1998.

[3] W. Whitt, "Blocking when service is required from several facilities simultaneously," *AT&T Technical Journal*, vol. 64, pp. 1807–1856, 1985.

[4] Keith W. Ross, *Multiservice Loss Models for Broadband Telecommunication Networks*, Springer, 1995.

[5] Debasis Mitra and John A. Morrison, "Erlang capacity and uniform approximations for shared unbuffered resources," *IEEE Transactions on Networking*, vol. 2, pp. 558–570, 1994.

[6] A.Dan, D.Sitaram, and P.Shahabuddin, "Scheduling policies for an on-demand video server with batching," in *Proceedings of ACM Multimedia 94 Conference*, Oct. 1994, pp. 15–23.