Analysis of Carrier Sensing’s Influence on the Performance of Routing Protocols in Multi-hop, Multi-rate Wireless Networks

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Abstract—Carrier sensing has been used as an effective way to reduce collisions and exploit spatial reuse in wireless networks. Previous research has attempted to tune the carrier sensing range to maximize the network throughput. However, the impact of carrier sensing threshold on the probability of successful transmission has been ignored. In this paper, we derive an analytical model to calculate the successful transmission probability. We then calculate the throughput of routing protocols using different link metrics. To the best of our knowledge, this perhaps is the first attempt to derive the throughput of routing protocols like expected transmission count (ETX) [1] and expected transmission time (ETT) [2] in wireless networks. We also investigate the impact of some other important factors, such as node density, average contention window size and packet length. Our results show that optimal routing protocols that are using ETX as the path metric can achieve around 30% more throughput than those using ETX and End-to-end delay. Compared to the minimum hop count protocols such as DSR and AODV, the optimal routing protocol can improve the throughput by up to 100%.

I. INTRODUCTION

Due to the shared medium and distance attenuation properties of wireless communications, efficient medium access control (MAC) protocols are difficult to design. In IEEE 802.11 [3], a CSMA/CA-based MAC has been defined to provide the medium access. Carrier sensing, which is the fundamental mechanism of the CSMA/CA protocol, reduces the likelihood of collisions by preventing stations within a certain range of the transmitter from transmitting. Too conservative a value of the carrier sensing threshold (CSth), on the other hand, may reduce the number of concurrent transmissions, which is referred to as “spatial reuse”. Thus, in order to balance spatial reuse and transmission protection, there likely exists an optimal carrier sensing threshold which can achieve the best network performance.

Many papers [4]–[11] have already attempted to optimize the system throughput by tuning carrier sensing range and transmit power. Yang and Vaidya [7] shows that the MAC overhead, bandwidth-independent and bandwidth-dependent, has a significant effect on the choice of carrier sensing range. Zhai and Yang [8] identify the optimum carrier sensing range for different data rates. They show that for the same model as in [7], the optimal carrier sensing range for different data rates varies very little. Lin and Hou [10] consider both the carrier sensing and back-off procedure to calculate the aggregate interference. But they did not take into consideration the next-hop selection of routing protocols.

In this paper, we first use a simple discrete Markov Chain model to derive the transmission attempt probability of each station. Then we calculate the successful delivery ratio of each station senses the channel and the received signal exceeds

II. ANALYTICAL MODEL

In this section, we first describe the propagation and interference model. Then we present our assumptions for the system. The probability that a station attempts to transmit is derived. We then calculate the successful transmission probability, and one-hop throughput for different routing protocols.

A. Interference Model

In our analytical model, the propagation model we use only considers the distance attenuation due to path loss. We neglect multi-path fading, or fading due to obstacles. Let $P$ denote the transmit power, $d$ the distance between the transmitter ($TX$) and receiver ($RX$), and $\gamma$ the path loss coefficient which ranges from 2 to 4. The received power $P_R$ at the receiver can be written as:

$$P_R = P \times \left( \frac{1}{d} \right)^\gamma$$ (1)

Let $CS_{th}$ denote the carrier sensing threshold. When a station senses the channel and the received signal exceeds
the carrier sensing range based on the propagation model:
\[
d_{cs} = \left( \frac{P}{CS_{th}} \right)^{\frac{1}{\gamma}}
\]  \hspace{1cm} (2)

Stations that are farther than \(d_{cs}\) away from the transmitter can transmit at the same time. However, in the worst case, this allows six 1st tier stations which are \(d_{cs}\) away to initiate transmissions at the same time as the transmitter, as shown in Fig. 1. This scenario is used to calculate the network throughput in [7]–[9]. The worst case is too conservative as there may not be stations at these precise positions. In addition, the probability that they transmit at the same time is very small.

**B. System Assumptions**

In order to simplify the analysis, we make some assumptions:

1) Nodes are randomly distributed on a 2-D plane according to a Poisson distribution with density \(\lambda\). In an area \(S\), the probability of there being \(n\) stations is:

\[
p_n = \frac{(\lambda S)^n}{n!} e^{-\lambda S}
\]  \hspace{1cm} (3)

The average number of stations in area \(S\) is \(\lambda S\).

2) Although the optimal system throughput could be achieved by tuning the carrier sensing threshold and transmit power together, we assume that all stations in the network use fixed transmit power.

3) The network is in saturated mode. Stations always have packets for transmission. We also assume that the packet size is fixed for all stations.

4) The receiver can decode the packet correctly if the SINR (signal to interference and noise ratio) at the receiver exceeds the minimum required SINR:

\[
SINR = \frac{P/d^\gamma}{\sum_{i \in \{ \text{arg } TX_i \neq TX X \}} P/d_i^\gamma} \geq \beta(r)
\]  \hspace{1cm} (4)

Where \(\beta(r)\) is the minimum SINR required to decode the packet sent by the transmitter using data rate \(r\).

**C. Transmission Attempt Probability**

The status of any station in the network can be modeled as a simple discrete Markov Chain, as shown in Fig. 2. There are three possible states. Stations may be transmitting or waiting for the channel to be idle. The third state is when the channel is idle and stations are counting down their back-off numbers.

The transition probabilities from Transmit and Wait state to Idle are both 1. When a station is not transmitting and the perceived signal is below \(CS_{th}\), the channel is considered to be idle. If the channel is idle, a station will initiate a transmission only when the back-off counter reaches 0 which, as derived by Cali [12], averages \(\frac{2^{cw} - 1}{cw}\), where \(cw\) is the average congestion window.

The Markov Chain model only looks at the system at the beginning of time-slots. The state probabilities are not affected by the durations of different time-slots. Let \(p_{tx}\), \(p_{idle}\) and \(p_{wait}\) denote the probabilities of the states in the Markov Chain model. Let \(p_{tx}, p_{idle}\) and \(p_{wait}\) denote the probabilities when we observe the system at any time instant. The relations between these two sets of probabilities are as below:

\[
p_{tx} = \frac{P_{tx} \times T_{tx}}{P_{tx} \times T_{tx} + P_{idle} \times T_{idle} + P_{wait} \times T_{wait}},
\]  \hspace{1cm} (5)

where \(T_{tx}\), \(T_{idle}\) and \(T_{wait}\) denote the durations of a transmission, one idle time-slot and the time spent waiting, respectively.

As shown in Fig. 3, we divide the space into rings. The step size is \(\Delta d\), which is picked to be small enough so that there is at most one station in each ring. Let \(p_d\) represent the probability that one station exists in the ring at distance \(d\). We then have:

\[
p_d = \lambda \Delta S \times e^{-\lambda \Delta S},
\]  \hspace{1cm} (6)

where \(\Delta S=2\pi d \Delta d\). If this station does exist, the interference it introduces is:

\[
I_d = \left( \frac{d_{cs}}{d} \right)^\gamma CS_{th}
\]  \hspace{1cm} (7)

Suppose that \(X_d \in \{0, 1\}\) is a Bernoulli random variable with mean \(p_d \times p_{tx}\), then the interference from the possible stations in the ring can be written as \(I_d X_d\). Let \(X\) represent the interference from all other stations at the transmitter, we have:

\[
X = \sum_{d = k \Delta d, k=1,2,3...}^{\infty} I_d X_d
\]  \hspace{1cm} (8)

Since stations that are far away will introduce very small interference, we can neglect stations farther than a specific distance, which we pick to be 5\(d_{cs}\). Eq. (8) can be simplified to:

\[
X = \sum_{d = k \Delta d, k=1,2,3...\left[ \frac{5d_{cs}}{\Delta d}\right]}^{\infty} I_d X_d
\]  \hspace{1cm} (9)
Then the probability that the channel is sensed to be busy, $P_{bus}$, can be expressed as:

$$P_{bus} = P\{X \geq CS_{\text{th}}\} \quad (10)$$

The channel is idle for a station only when itself is not transmitting and the channel is not busy, which occurs with probability:

$$p_{idle} = (1 - p_{tx})(1 - P_{bus}) \quad (11)$$

Also, from the state transit diagram, we have:

$$P_{tx} = P_{idle} \cdot \frac{2}{cw + 1} \quad (12)$$

By plugging Eq. (5)-(11) into Eq. (12), we can calculate the transmission probability for each station in the network.

D. Successful Transmission Probability

Let $X'$ represent the interference at the receiver. $X'_d \in \{0, 1\}$ represents the event that there is a station transmitting in the ring at a distance $d$ away from the receiver with interference power $I'_d$. We then have:

$$X' = \sum_{d=k\Delta d, k=1,2,3...} I'_d X'_d \quad (13)$$

Let $I_{\text{tolerate}}$ denote the interference that the receiver can tolerate to decode the data packet successfully. The transmission will be successful only when the interference is smaller than $I_{\text{tolerate}}$, which is:

$$P_{\text{suc}} = P\{X' < I_{\text{tolerate}}\} \quad (14)$$

A ring may have two parts: an area within carrier sensing range and an area outside the range, as denoted by $S_A$ and $S_B$, respectively, in Fig. 4. Stations in these two areas have different probabilities of transmitting at the same time as the transmitter $TX$. Let $P_{tx}^A$ and $P_{tx}^B$ represent the probabilities that stations in $S_A$ and $S_B$ transmit concurrently as $TX$. The expected value of $X'_d$ then can be written as:

$$E(X'_d) = p_d \times (P_{tx}^A \frac{S_A}{S_A + S_B} + P_{tx}^B \frac{S_B}{S_A + S_B}) \quad (15)$$

where $p_d$ is the probability that one station exists in the ring at distance $d$, which is given in Eq. (6).

Transmissions of stations in $S_A$ may overlap with $TX$’s only when their back-off counters reach 0 at the same time, which is:

$$P_{tx}^A = \frac{2}{cw + 1} \quad (16)$$

Fig. 3: Interference Rings

Fig. 4: Ring Overlap

Stations in area $S_B$ are out of the carrier sensing range. Unlike stations in $S_A$, they may transmit before or after $TX$ starts transmission. Suppose that there is a station $N_B$ in $S_B$. We define the events below:

- $B_1 = \{N_B$ is transmitting when $TX$ starts its transmission$\}$
- $B_2 = \{\text{Medium appears idle to } N_B \text{ when } TX \text{ starts its transmission, and } N_B \text{ starts to transmit before } TX \text{ finishes its transmission}\}$
- $B_3 = \{N_B \text{ is waiting for the channel to be idle when } TX \text{ starts its transmission, and } N_B \text{ starts to transmit before } TX \text{ finishes its transmission}\}$

As derived in Sec. II-C, $P(B_1) = p_{tx}$. If $N_B$ is not transmitting at the moment $TX$ initiates its transmission, $N_B$ will stay in its current state for, on average, half of the time-slot which is $T_{idle}/2$ or $T_{\text{wait}}/2$. Let $P_{mn}^B$ denotes the probability that $N_B$ will stay in $\text{Idle}$ state for $m$ slots and $n$ slots in $\text{Wait}$ state before transmitting. It can be calculated as:

$$P_{mn}^B = (P_{\text{tx}})^m \times (P_{\text{2w}})^n \times \frac{2}{cw + 1} \quad (17)$$

$P_{\text{tx}}$ and $P_{\text{2w}}$ are the transition probabilities from State $\text{Idle}$ to $\text{Idle}$ and from State $\text{Idle}$ to $\text{Wait}$ respectively. They can be calculated from the transmission attempt probability we derived in Sec. II-C. So we can get the probabilities of events $B_2$ and $B_3$:

$$P(B_2) = p_{\text{idle}} \sum_{m,n \text{ s.t. } (m+\frac{1}{2})T_{\text{idle}} + nT_{\text{wait}} < T_{\text{tx}}} P_{mn}^A \quad (18)$$

$$P(B_3) = p_{\text{wait}} \sum_{m,n \text{ s.t. } mT_{\text{idle}} + (n+\frac{1}{2})T_{\text{wait}} < T_{\text{tx}}} P_{mn}^A \quad (19)$$

Then the probability that $N_B$’s transmission overlaps with $TX$’s can be derived:

$$P_{tx} = P(B_1) + P(B_2) + P(B_3) \quad (20)$$

By plugging Eq.(16) and (20) into Eq.(15), we can calculate the expected value of the Bernoulli random variable $X'_d$.

Then we can calculate the probability that the interference is below the tolerable level, which is the successful transmission probability.

E. Next-Hop Selection of Different Routing Protocols and Their Performance

Suppose that a station has one packet to forward to a destination at distance $d$. As we have derived, the probability of successful transmission is $P_{\text{suc}}(d)$. On average, station has
to send a packet \(1/P_{\text{suc}}(d)\) times for the destination to decode the packet correctly. We define a gain \(G\) which is quite similar to the bandwidth distance product (BDiP) proposed in [8].

\[
G = d \times P_{\text{suc}}(d) \times r_d, \tag{21}
\]

where \(r_d\) is the data rate, which can be derived by:

\[
r_d = \frac{L_{pk}}{T_{PHY} + \frac{L_d + L_{pk}}{r_{rx}} + T_{ACK} + SIFS + DIFS}, \tag{22}
\]

where \(L_{pk}\) and \(L_d\) are lengths of data and MAC header, respectively. \(T_{PHY}\) and \(T_{ACK}\) are the time used to transmit the physical header and one acknowledgement packet. \(SIFS\) and \(DIFS\) are the inter-frame spaces. \(r_{tx}\) is the transmission data rate.

In Fig. 5, we show the transmission gain (G) at different data rates, 6Mbps, 18Mbps, 36Mbps and 54Mbps of IEEE 802.11g protocol. The node density is 0.001 per square meter and the carrier sensing range is 150m, while the transmission range at 6Mbps is 100m. In the following, we will first analyze how different routing protocols select the next hop and calculate their one-hop throughput.

1) Minimum Hop-Count Routing Protocols (MHCR): As shown in Fig. 6, we divide the neighbors of a station into 4 directions. Only stations in the direction pointing to the final destination can be selected as the next hop. It is possible that in a very sparse network, stations in other directions can achieve a better performance as the next hop. But we neglect that case. We divide the transmission range \(d_{tr}\) into segments of length \(\Delta d\), so that the transmission range can be viewed as circles with radius \(\Delta d\), \(2\Delta d, 3\Delta d, \ldots \), \(\Delta d\) is small enough that in area \(\frac{1}{2} \pi \Delta d \Delta d\), there can be at most only one station. The probability that one station exists there is \(1 - e^{-\frac{1}{2} \pi \Delta d \Delta d}\).

In routing protocols like AODV, the farther stations have higher priority. But we consider that in reality the stations may miss the route setting up packet Route Request (RR) which is broadcast using the lowest data rate. Let \(P_{\text{MHC}}(d)\) denote the probability that a station exists at distance \(d\) away and decodes the RR packet successfully so that it can be a candidate for the next hop. We have:

\[
P_{\text{MHC}}(d) = P_{\text{suc}}(d)(1 - e^{-\frac{\Delta d \Delta d}{2}}) \tag{23}
\]

Let \(P_{\text{MHC}}(d)\) denote the probability that a station at distance \(d\) is selected as the next hop, we have:

\[
P_{\text{sel}}(d) = P_{\text{MHC}}(d) \cdot \prod_{d_i = d + \Delta d} d_{tr} \left(1 - P_{\text{MHC}}(d_i)\right) \tag{24}
\]

The one-hop throughput (OHT) then can be expressed as:

\[
OHT = \lim_{\Delta d \to 0} \sum_{i=1}^{d_{tr}/\Delta d} \Delta d \times r_i \Delta d \times P_{\text{suc}}(i\Delta d) \tag{25}
\]

2) Minimum Expected Transmission Count Routing Protocols (METXR): ETX as proposed in [1] different from MHCR in that takes the link loss rate into account. The link metric used is the expected transmission times over one link. It is indirectly using the expected advanced distance (EAD) as the link metric, which does not consider the transmission data rate:

\[
EAD = d \times P_{\text{suc}}(d) \tag{26}
\]

In this case, the probability that a station at distance \(d\) away would be selected as next hop is:

\[
P_{\text{ETX}}(d) = P_{\text{ETX}}(d) \cdot \prod_{EAD(d_i) > EAD(d)} \left(1 - P_{\text{ETX}}(d_i)\right) \tag{27}
\]

3) Minimum End-to-End Delay Routing Protocols (MEDR): In protocols that consider the multi-rate support in wireless networks like IEEE 802.11, end-to-end delay is used. They select the paths that have the minimum cumulative transmission time over all the hops, which effectively uses the BDiP (Bandwidth Distance Product) as the link metric.

\[
BDiP = d \times r_d \tag{28}
\]

Thus, a station at distance \(d\) has the following probability of being selected:

\[
P_{\text{MED}}(d) = P_{\text{MED}}(d) \cdot \prod_{BDiP(d_i) > BDiP(d)} \left(1 - P_{\text{MED}}(d_i)\right) \tag{29}
\]

An equation similar to Eq. (25) then can be used to calculate the one-hop throughput.

4) Minimum Expected Transmission Time Routing Protocols (METTR): Routing protocols using ETT as the link metric can achieve the maximum throughput because it also takes the link loss rate into consideration. As we have shown in Fig. 5, stations at different distances have different optimal transmission gain. Those stations with more transmission gain have higher priority to be selected. Thus, we have the probability of next-hop selection:

\[
P_{\text{ETT}}(d) = P_{\text{ETT}}(d) \cdot \prod_{TG(d_i) > TG(d)} \left(1 - P_{\text{ETT}}(d_i)\right) \tag{30}
\]
The impacts of these factors on the network performance, and Table I. The transmission range is the farthest distance that area. Data rates, SINRs and transmission ranges are shown in different routing protocols. We illustrate the throughput of density and carrier sensing threshold on the performance of different node densities. As shown in Fig. 7, in a very dense network, the throughput increases with the contention window size. But in a sparse network, choosing a smaller window size leads to a more efficient use of channel. The analysis above shows that system throughput of a routing protocol is a function of node density $\lambda$, average contention window size $cw$, packet length $L_{pk}$ and carrier sensing threshold $CS_{th}$. In the next section, we will study the impacts of these factors on the network performance, and how different routing protocols perform in different scenarios.

III. MODEL VALIDATION

In this section, we carry out simulation studies to validate our analytical model. We use the OPNET simulator to study the impact of contention window size, packet length, node density and carrier sensing threshold on the performance of different routing protocols. We illustrate the throughput of different routing protocols in different scenarios. We also show how much more gain the METTR and METXR routing protocols perform than protocols like MHCR and MEDR.

A. Simulation Setup

Stations are randomly placed in a $1000m \times 1000m$ square area. Data rates, SINRs and transmission ranges are shown in Table I. The transmission range is the farthest distance that

\begin{equation}
T_{tx} = T_{wait} = \lim_{\Delta d \to 0} \sum_{i=1}^{d_{tx}/\Delta d} P_{sel}(i\Delta d) \times T_{tx}(i\Delta d)
\end{equation}

where $T_{tx}(i\Delta d)$ is the transmission time when a station at $i\Delta d$ is selected as the next-hop. It can be computed as:

\begin{equation}
T_{tx}(d) = \frac{L_{pk}}{T_{PHY} + \frac{L_{PHY} + L_{pk}}{r_{tx}(d)} + SIFS + DIFS + T_{ACK}}
\end{equation}

where $r_{tx}(d)$ is the data rate that can achieve the most transmission gain (G) when the next hop is $d$ units away.

The throughput for one station (User Throughput) in the network can be written as:

\begin{equation}
UT = \frac{P_{tx} \times OHT}{P_{tx} \times T_{tx} + P_{idle} \times T_{idle} + P_{wait} \times T_{wait}}
\end{equation}

The system throughput (ST) is the sum of the throughputs of all users in the network, which is:

\begin{equation}
ST = \lambda S \times UT
\end{equation}

The analysis above shows that system throughput of a routing protocol is a function of node density $\lambda$, average contention window size $cw$, packet length $L_{pk}$ and carrier sensing threshold $CS_{th}$. In the next section, we will study the impacts of these factors on the network performance, and how different routing protocols perform in different scenarios.

B. Impact of Contention Window Size

In order to examine the impact of contention window size on the network performance, we use fixed contention window size. We set $CS_{th}$ to be $150m$, and the packet size to 512 Bytes. We ran simulations for scenarios with different node densities. As shown in Fig. 7, in a very dense network, the throughput increases with the contention window size. But in a sparse network, choosing a smaller window size can maximize the throughput. As shown in Fig. 7(c), this optimal $cw$ is around 200. The reason is that with a larger contention window, a station has a smaller probability to transmit, which will reduce both collisions caused by stations within the carrier sensing range and interference from stations outside $d_{cs}$. In a dense network, a larger window size can protect the transmission. But if the network is not that dense, a smaller window size leads to a more efficient use of channel.

C. Impact of Packet Size

Fig. 8 depicts the impact of packet size on the network throughput and the optimal carrier sensing range. The node density of the simulations is $\lambda = 4 \times 10^{-4}$, and the packet sizes are 500 Bytes, 1000 Bytes and 1500 Bytes respectively. One important observation is that larger packet size leads to smaller network throughput. This is because when a station sends a packet to a destination $d$ units away, a longer packet length need longer transmission time and thus has a higher probability of being received in error due to aggregate interference from stations outside the carrier sensing range. On the other hand, a shorter packet size results in much more overhead. So there is an optimal packet size given the node density. When

<table>
<thead>
<tr>
<th>Rates (Mbps)</th>
<th>SINR (dB)</th>
<th>Transmission Range (m)</th>
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<tbody>
<tr>
<td>6</td>
<td>6.02</td>
<td>250.00</td>
</tr>
<tr>
<td>9</td>
<td>7.78</td>
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<tr>
<td>12</td>
<td>9.03</td>
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<td>18</td>
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<td>54</td>
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In this paper, we investigate the interactions between the MAC-layer parameters and the routing-layer next-hop selection. The network throughput depends strongly on the link metrics used for path selection. We derive that the throughput is a function of node density $\lambda$, average contention window size $cw$, packet length and carrier sensing threshold $CS_{th}$. We calculate the network throughput for some popular routing protocols that use different link metrics. Our analytical and simulation results agree quite well. The results can give us some insight into future network design, for parameters such as rate adaptation, contention window size and packet fragmentation threshold.

IV. CONCLUSIONS

In this paper, we investigate the interactions between the MAC-layer parameters and the routing-layer next-hop selection. The network throughput depends strongly on the link metrics used for path selection. We derive that the throughput is a function of node density $\lambda$, average contention window size $cw$, packet length and carrier sensing threshold $CS_{th}$. We calculate the network throughput for some popular routing protocols that use different link metrics. Our analytical and simulation results agree quite well. The results can give us some insight into future network design, for parameters such as rate adaptation, contention window size and packet fragmentation threshold.

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